Radio Wave Propagation: How Waves Attenuate with Distance

Inverse square law attenuation holds for free space — but attenuation can be stronger in many environments.

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I operate a 5 W VHF handheld transceiver, and with it I can talk to hams orbiting 400 km (249 miles) above Earth in the International Space Station (ISS). Why can't I talk directly to another handheld radio just 10 km (6 miles) away across town? The direct path to ISS is free from obstructions, so signals attenuate according to the free space inverse square law. Near the ground however, signals can travel by multiple paths with many signal copies arriving delayed in time.

This directly affects the rapidity with which signals attenuate with distance. That is, the exponent of the propagation inverse power law increases compared to inverse square law. So where does the energy go? Transmitted energy spreads in two dimensions (area) as it travels in the third dimension (distance), but the energy can also spread in the fourth dimension (time) because of multipath scattering. We'll apply different models to describe propagation in the various environments, and we will look into a few simple theoretical models to explain the underlying principles.

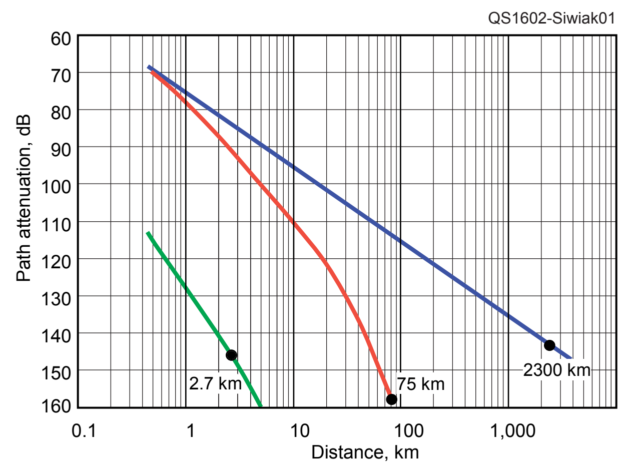

Figure 1 — Two 5 W VHF transceivers can talk 2300 km to the ISS on a clear line-of-sight path (blue), or 75 km to a tall repeater antenna (red), but less than 3 km (green) from street level to street level.

The VHF Signal Path

Let's look closely at a pair of handheld transceivers. Each radio transmits 5 W or +37 dBm (decibels relative to a milliwatt), and receives with a typical sensitivity of 0.16 µV (–123 dBm). The difference between the transmitter power level and the receiver sensitivity is 37+123=160 dB. That's how much *link margin* we can "burn up" in the path, including the antenna gain or loss. Said otherwise, you can place a 160 dB attenuator between the antenna connectors of the two transceivers, and they will communicate at the limits of performance. We can include a typical –7 dBi gain "rubber ducky" antenna at each end, so $160 + 2 \times (-7) = 146$ dB path link margin remains. How far will 146 dB take you at VHF? That depends on the path-specific propagation law.[1]

Free Space Propagation

The path distance to ISS can vary from 400 km (249 miles) straight up, to about 2300 km (1429 miles) on the horizon. The

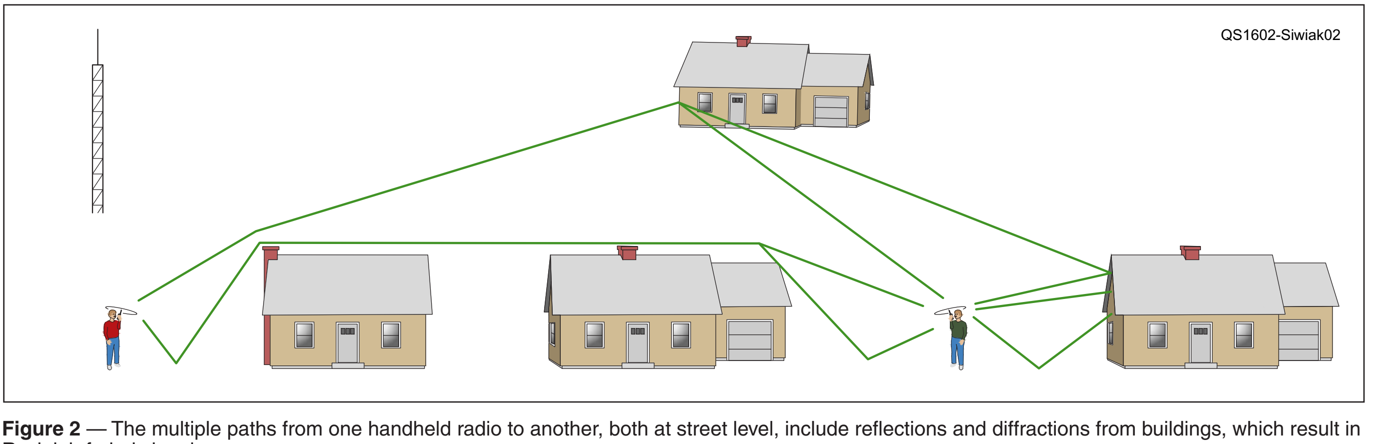

Figure 2 — The multiple paths from one handheld radio to another, both at street level, include reflections and diffractions from buildings, which result in Rayleigh faded signals.

unobstructed path is line-of-sight, so the radiated power density diminishes with the square of the distance, as explained by Joel Hallas, W1ZR.[2] The free space path attenuation in decibels is

$$P_{FS} = 32.4 + 20\log\left(F_{MHz} D_{km}\right) \qquad \text{[EQ 1]}$$

Plug in F_{MHz}=146, and D_{km}=2300 and we get a path attenuation of P_{FS}=143 dB, or a 3 dB stronger signal than we need at the horizon for two 5 W handheld transceivers! Of course, ISS uses antennas that are somewhat better than a rubber ducky, and the antenna polarizations must be aligned. We've ignored ground reflections, which affect the signal near the horizon. But you get the basic idea — in the absence of interference, you can reach ISS with a 5 W handheld transceiver. Figure 1 shows the path attenuation (blue) to ISS — the slope of the attenuation is 20 dB per decade of distance, or inverse square law.

The Cross-Town Suburban Path

Then why can't I talk very far across town to another handheld transceiver? The radiated energy still expands spherically, but there are additional factors when both transceiver antennas are below the suburban or urban building roof lines. The waves (see Figure 2) illuminate the local buildings and streets, and travel by multiple paths. When they reach building roof lines, or building corners, they diffract — this adds diffraction losses. They then propagate further by multiple paths. The green curve in Figure 1 shows the signal attenuation for the cross town path.[3] Diffraction losses, and losses along the rooftops of the suburban path in Figure 2, add a further 62 dB of loss on top of the free space loss at a distance of 2.7 km (1.6 miles) between handheld transceivers, for a total of 146 dB attenuation! However, lots of that energy diffracts upwards, above the rooftops. That's where repeaters show their stuff — repeaters elevate one end of the link far above the rooftops for everyone.

Repeaters in Urban Paths

Everyone talks and listens to the repeater, as seen in Figure 3. One end of the path is high above the suburban buildings, while the other end might still be at street level. That part of the link is subjected to scattering and diffraction losses from the nearest roof edge.

A repeater antenna on a 61 m (200 ft) tower with a +6 dBi gain antenna — 13 dB more than the handheld transceiver antenna — has a path link margin of 159 dB to a handheld transceiver. A suburban propagation model predicts (red curve in Figure 1) that the signal attenuates initially at 30 dB per decade of distance (inverse third power law), then attenuation becomes somewhat stronger as the distance to the repeater tower nears the horizon.[4] So, we've beat the diffraction losses at one end of the link, but the path law is still more severe than a free space path. Furthermore, we receive multiple time-delayed copies of the signal, which results in Rayleigh fading.[5]

Transmitted energy spreads in two dimensions (area) as it travels in the third dimension (distance), and the energy can also spread in the fourth dimension (time) because of multipath scattering.

Multipath Propagation

Energy transmitted in open air expands spherically, yet we observe that along suburban paths it can diminish faster than by inverse square law. No, we don't violate the *Law of Energy Conservation*. So where does the excess energy go? The "missing" energy can go into several places. First, some energy can be lost to dissipative heating, but we'll concentrate on the energy that is not dissipated. Second, some of the energy spreads and expands, encountering reflectors (like the ground) and scatterers. So a large fraction of the energy can be *redirected* somewhere else, usually up. An example of this is a two-path scenario that results in an *inverse fourth power propagation law*. Energy also scatters into the orthogonal polarization, but we'll not consider that here. Finally, in non-line of sight paths with dense scattering, the multipath energy gets redirected into the fourth dimension — time!

Two-Paths Near the Ground

Two-path propagation occurs between two antennas that are close to the ground, but otherwise in view of each other (see Figure 4). This is a common scenario on an OATS (open air test site) used for measuring antennas, and is a perfect example of inverse fourth power propagation between two handheld transceivers. Energy travels along a direct path, but also along a second path that reflects from the ground at a shallow angle of incidence. The ground refection coefficient is very nearly minus one at a shallow angle of incidence for almost any ground parameters. That means the direct path energy will cancel the reflected energy — except that the direct and reflected path lengths are slightly different. That slight difference in path lengths results in

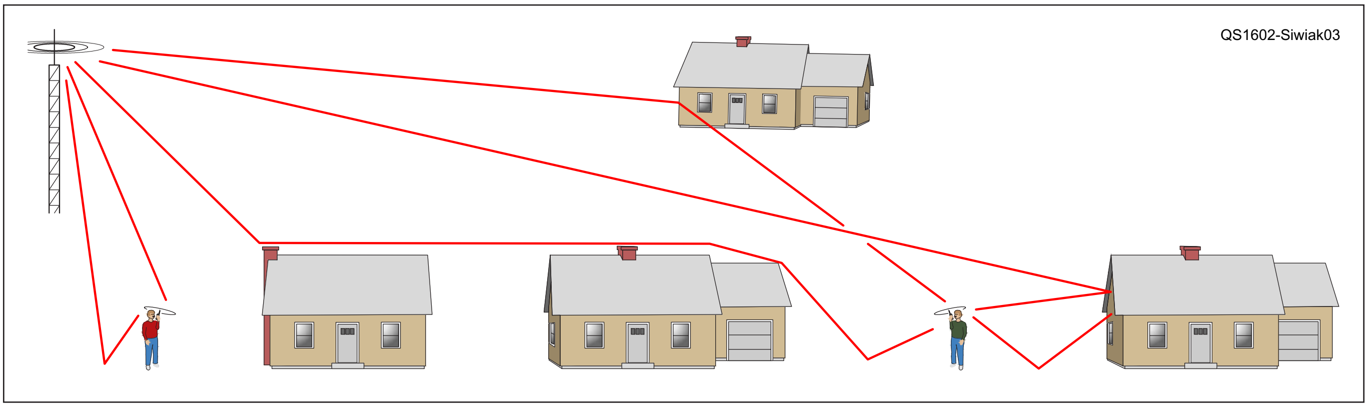

Figure 3 — A repeater elevates one end of the link above local rooftops, but the street level end still encounters local reflections and diffractions from buildings, which result in Rayleigh faded signals.

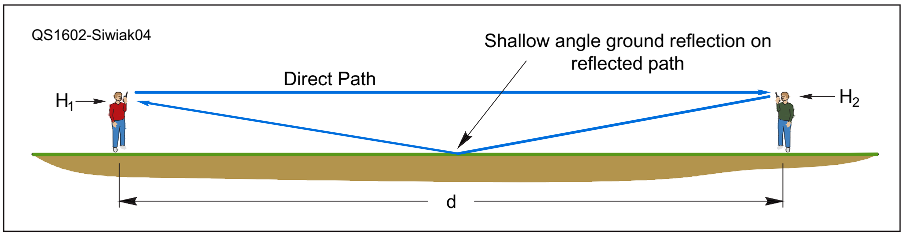

Figure 4 — This open air test site scenario includes a direct path and a path that reflects from the ground at a very shallow angle. As distance increases, the path attenuation follows an inverse fourth power law.

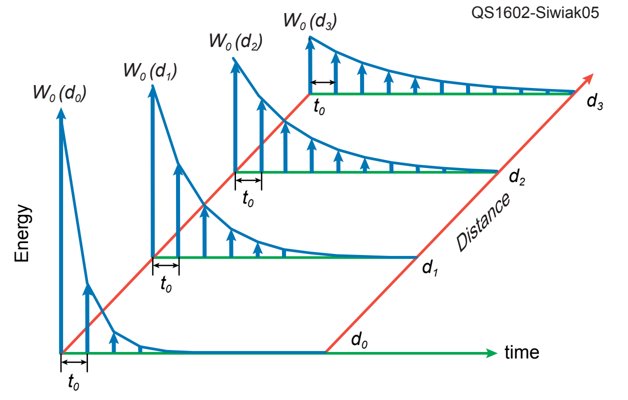

Figure 5 — As energy travels along the distance axis (red) *d*, increasingly more energy is time-delayed and shed into the time axis (green).

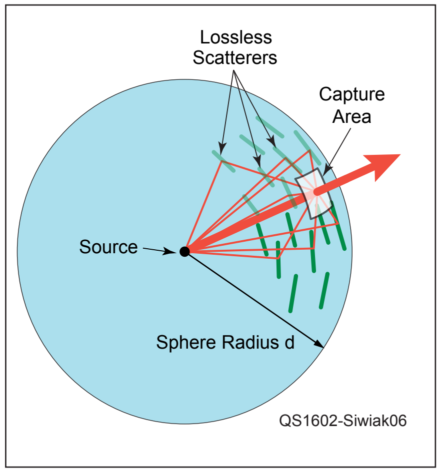

Figure 6 — Total energy over the full spherical surface equals the source energy. On average, the energy in the capture area attenuates with inverse square law. The scatterer density is uniform, so as the radius *d* of the sphere gets bigger, the number of scatterers involved in multipath — and the delay spread — also increases.

an imperfect phase cancellation of the two waves. We can easily show that the path attenuation for this case simplifies to

$$P_{2-path} = \frac{(H_1 H_2)^2}{d^4} \qquad \text{[EQ 2]}$$

for distances d greater than about $4\pi H_1 H_2/\lambda$, (20 meters or 67 feet for VHF radios at head level). For closer distances, the attenuation fluctuates around the free space value. H_1 and H_2 are the two handheld radio heights above the ground and λ is the wavelength, all in the same units. Notice that the exponent for d is four — hence inverse fourth power law — and also that Equation 2 is independent of frequency! This ground reflection also explains the null at very low elevation angles in HF propagation from elevated antennas.

Multiple Paths

Delay spread is a statistical measure of the time delays among the different paths connecting the two antennas in Figures 2 and 3. "Interpretation of some measured data indicates that an exponential distribution of the delay spread is a good approximation."[6] Delay spread τ_d is the "delay factor" in that exponential distribution.

Figure 5 illustrates a model of multipath signals represented by energy impulses traveling along distance d. At $d=d_0$ the strongest impulse $W_0(d_0)$ is followed by a few impulses delayed by multiples t_0 of time. By the time the energy reaches distance d_3, the strongest impulse $W_0(d_3)$ is followed by a much longer trail of exponentially decaying impulse amplitudes. These distances d_{0-3} are representative of different paths in Figures 2 and 3. So not only does the total energy — the sum of all the impulses at a given distance — attenuate with the free space inverse square law, but the leading impulse also further attenuates by shedding energy into more and more paths as distance increases!

The full picture of the propagation of energy $W(d)$, and what is going on with the impulses in Figure 5 can be represented by the infinite summation in Equation 3. Bear with me and don't get glassy-eyed yet — we will greatly simplify this formula.[7]

$$W(D_{km}) = \left[\frac{1}{(42 F_{MHz} D_{km})^2}\right] \times \left(1 - \exp\left[\frac{-t_0}{\tau_d}\right]\right) \sum_{n=0}^{\infty} \exp\left(\frac{-n t_0}{\tau_d}\right) \qquad \text{[EQ 3]}$$

Distance d is D_{km} in kilometers to match the units in Equation 1. $W(D_{km})$ is the total path loss for the sum total of all the multipath components. The first term in brackets is the free space propagation law, which we stated as decibels of attenuation in Equation 1. The remaining terms include *all* of the multipath energy terms, which add up to exactly one!

If we had a *rake* receiver — a type of diversity receiver that can lock onto and correctly align all of the multipath components — we would experience just free space attenuation in this otherwise lossless multipath scenario. We don't, so *at best* our conventional ham receiver will lock to the strongest or the W_0 term corresponding to n=0 in Equation 3. That is the leading impulse in Figure 5. The rest of that infinite summation represents energy that goes into Rayleigh fading.

Figure 6 shows how energy expands spherically from a central source. Let's uniformly distribute many lossless and co-polarized scatterers inside the sphere. We can easily see that the total energy on the sphere at any radius equals the energy emitted by the source. However, as the size of the sphere increases, the number of scatterers involved in multipath also increases, so the delay spread also increases with distance.

Time and distance in propagation are related by the speed of light c, so in

Figure 5, distance $d{=}ct$. We know from measurements that delay spread τ_d increases with distance at an exponential rate M, so we can replace t_0/τ_d with $(D_B/D_{km})^M$, where D_B is a *breakpoint distance* at which the delay spread τ_d takes effect, and M is an environment-dependent exponent.[8] Measurements suggest that M is typically between 0.5 and 2.5, and that D_B is roughly the distance to the nearest scatterers and can be as low as a few meters indoors to hundreds of meters outdoors. The path loss, considering just the strongest impulse, reduces to

$$W_0(D_{km}) = \frac{1-\exp\left(\left[\dfrac{-D_B}{D_{km}}\right]^M\right)}{\left(42F_{MHz}D_{km}\right)^2} \qquad \text{[EQ 4]}$$

We end up with the free space term (denominator) modified by a distance dependence (numerator) that is related to delay spread. $W_0(d)$ starts out with inverse square law, then at distances D_B it transitions to an inverse $(2{+}M)$ power law. When the distance D_{km} greatly exceeds D_B, we can replace that entire numerator by $(D_B/D_{km})^M$. As promised, that eye-glazing Equation 3 simplifies to

$$W_0(D_{km}) \approx \frac{\left[\dfrac{D_B}{D_{km}}\right]^M}{\left(42F_{MHz}D_{km}\right)^2} \qquad \text{[EQ 5]}$$

The net path loss exponent on distance D_{km} is $(2{+}M)$. Simplicity! We now have a physical basis for understanding path attenuation that falls off faster than the free space law *without considering dissipative losses*.

Conclusions

In propagation along multiple paths, many copies of the signal arrive delayed in time. Those time-delayed copies of the signal interfere with one another and cause multipath fading. The time-delayed multiple copies steal energy from the strongest signal path, increasing the attenuation by another 0.5 to 2.5 exponent to an inverse 2.5 to 4.5 power! Further details, including the propagation models used in this article, are available at the *QST* in Depth web page.[9]

Notes

[1]Several propagation models appear in: K. Siwiak and Y. Bahreini, *Radiowave Propagation and Antennas for Personal Communications, Third Edition*, Artech House, Norwood MA: 2007.

[2]J. Hallas, W1ZR, "Antenna Gain, Part III: How Much Signal Gets Received?" *QST*, Jan 2016, pp 45-48.

[3]R. Maciel, H.L. Bertoni, H.H. Xia, "Unified approach to prediction of propagation over buildings for all ranges of base station antenna height," *IEEE Transactions on Vehicular Technology*, Vol. VT-42, No. 1, pp 41 – 45, Feb 1993, equations given in Section 7.2 of note 1.

[4]Y. Okumura et al., propagation model using M. Hata's equations modified in Section 7.3 of note 1.

[5]Alan Bloom, N1AL, "VHF/UHF Mobile Propagation," *QST*, Aug 2006, pp 35 – 37.

[6]W.C. Jakes, *Microwave Mobile Communications*, American Telephone and Telegraph Co., 1974, reprinted: IEEE Press, Piscataway, NJ, 1993, p 50.

[7]"Everything should be made as simple as possible — but not simpler," attributed to Albert Einstein and others.

[8]K. Siwiak, H.L. Bertoni, and S. Yano, "Relation between multipath and wave propagation attenuation", *Electronic Letters,* 9 Jan 2003, Vol. 39 Num 1, pp 142 – 143.

[9]**www.arrl.org/qst-in-depth**

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